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AN ANALYSIS OF LAMINAR FREE CONVECTION AROUND ISOTHERMAL VERTICAL PLATE

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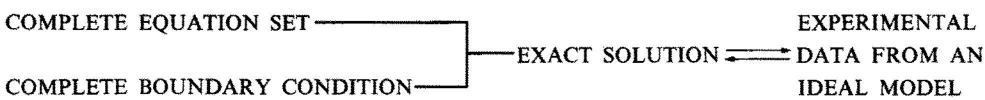
Abstract—Often in papers on natural convection the velocity and temperature distribution near a plane, vertical, isothermal surface are compared with theoretical predictions obtained from the solution of the differential equations for a semi-infinite vertical plate.

An attempt is made in the present work to explain the discrepancies emerging from such comparisons and to show that the lack of agreement observed is due to the conditions under which the experiments are run.

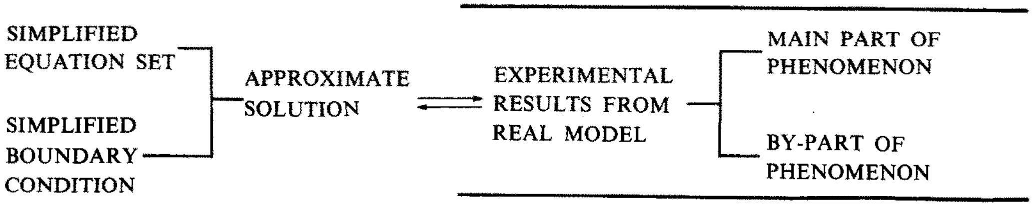
Particular attention has been given to the flow outside the boundary layer and the influence of the leading edge upon the flow inside the boundary layer.

1. INTRODUCTION

AN IDEAL comparison between results from a theoretical solution and experimental data is a comparison between the exact solution and experimental data from an ideal model.



Usually neither an exact solution nor an ideal experimental model are available. Because of this limitation of our information about the problem we are forced to compare the results obtained from approximate solution with the results from real experimental model.



The above system is somewhat general in its character and may be a subject to certain alterations depending on the nature of the problem in hand and the way it is tackled. This may be better explained by considering a specific example such as that discussed in the present work, namely natural convection on a vertical isothermal flat plate where the problem is reduced to the consideration of the velocity and temperature fields.

Experimental investigations are usually carried out on a vertical flat plate of finite dimensions, i.e. of finite thickness. The temperature and velocity distributions are measured close to the plate surface, the velocity measurements often being limited to the immediate vicinity of the plate surface and to their component in the direction along the plate. Although these values may be rightly considered as the most significant, a much wider zone is actually affected by the phenomenon, especially in the case of the velocity field. Moreover, the character of the velocity distribution contributes additional information. Accordingly, in the present work much attention has been given to the velocity field.

Laminar natural convection on an isothermal vertical plate is mathematically described by two appropriate solutions of simplified equations:

(1) the boundary-layer solution, valid in the immediate vicinity of a semi-infinite surface
(2) the first order perturbation solution, valid for a plate of a finite length and over a region much greater than that of the boundary layer. In this solution the boundary-layer solution is the zero-order approximation.

In the investigated case of natural convection the experimental results can be compared with solution (1) and for solution (2).

Discrepancies between experimental results and theoretical predictions have already been observed and analysed by several authors. Schmidt and Beckman [1] compared various empirical data with Pohlhausen's solution of the type (1). Again Eichhorn [2] compared his own measurements with solution (1) and the observed lack of agreement has been accounted for quantitatively in [3].

In his analysis Eichhorn suggested three causes for these discrepancies. The first one, namely the inadequacy of the mathematical description, has already been partly explained by the perturbation solution of Yang and Jerger [4]. The remaining two causes, namely: (1) the influence of the fluid motion outside the boundary layer and (2) the influence of the geometry of the leading edge, are explained in the present work.

2. CONVECTION FLOW AROUND A PLATE

Because in the present work the area considered is much wider than the traditionally studied boundary-layer zone, it will be useful to begin with giving a general description of the convection flow that takes place around a vertical plate.

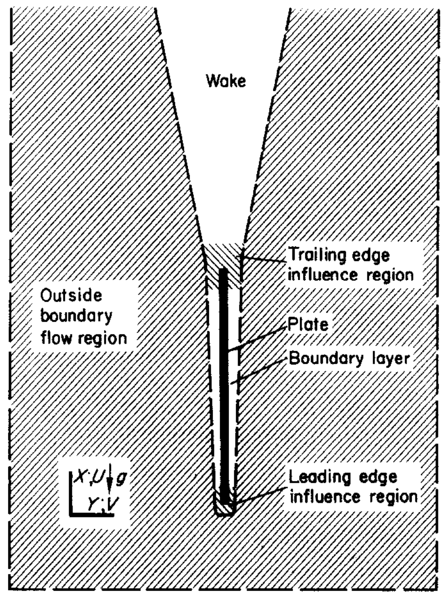


FIG. 1. Free convection process around a vertical isothermal plate.

The total area can be subdivided into several zones:

1. The boundary layer, characterized by the temperature and velocity fields typical of viscous flow. This can be further sub-divided into (a) the part of the boundary layer in the immediate vicinity of the flat surface of the plate. This part is the most important in the problem investigated and its velocity and temperature distributions are the main part of the phenomenon; (b) the part of the boundary layer near the leading edge of the plate where both the temperature and the velocity fields are influenced by the geometry of the edge; (c) the part of the boundary layer near the trailing edge of the plate.
2. The wake, an area above the trailing edge of

the plate, and in which the velocity and temperature fields are those of viscous flow.

3. An area surrounding the boundary layer and wake zones and in which the temperature is uniform and the flow may be treated as non-viscous.

It can be seen that the border lines between these sub-areas have a formal character and depend on the geometry of the plate and the velocity gradients to which the viscous forces are proportional.

The interaction between these sub-areas may be explained as follows: The air nearest to the plate warms up, decreases its density and moves upwards according to Archimedes law. Because of continuity the air outflow from this area causes an influx of air from the surroundings, that is from the sides and from below the plate. The flow inside the thin boundary layer thus affects the outside flow, at the same time any changes in the latter also cause changes within the boundary layer, and it has been observed experimentally that the inflow from the sides of the plate oscillates with time.

The flow from below depends on the geometry of the leading edge. Moreover at the leading edge free convection takes place on surfaces other than the vertical plane and heat conduction in the air gives rise to free convection upstream of the geometrical origin of the flat surface of the plate. Again, above the trailing edge of the plate convection is altered by the end effect and the wake, but the resulting disturbance is not likely to be propagated upstream to any significant degree and was therefore neglected.

3. DESCRIPTION OF THE METHODS AND THE EXPERIMENTAL DEVICES

As already mentioned, the main attention was directed towards the determination of the velocity field. The method employed in [3] was used, as being particularly suitable for the investigated problem. The velocity field was determined from the photographs of the trajectories of small dust particles. These particles were introduced to the space, in which the plate was placed, and were illuminated by stroboscope light. The trajectories of small dust particles can be considered as very closely approximating the streamlines.

The temperature field, treated in the present paper rather peripherally was determined by means of a Mach–Zehnder interferometer.

Three different plates were used in the course of the investigations. All of them 240-mm long (high) and 160-mm wide. The geometries of the leading edge is shown in the Fig. 2 which also shows the accepted coordinate system and the plate thickness in mm. Inside the plates, which were made of copper, there was a set of channels through which was flowed water from an ultra-thermostat. The proper design of these channels, their length and the high flux of water ensured constant temperature of the whole plate. Most of the tests of the influence of air motion in the outside zone on the flow in the boundary layer were carried out using No. 1 plate. This was designed to secure the symmetry of the process and both its surfaces and the leading edge were uninsulated. Number 2 and No. 3 plates were shaped to reduce to a minimum the influence of the leading edge. The inlet and outlet of the heating water were placed in the upper part of the plate, and were connected to the ultra-thermostat by means of well insulated pipes placed in the plane of the plate, thus eliminating the influence of their convection field on that of the test plate.

The gradient of the ambient temperature in the room in which the experiment was carried on was about 0·6 degC/m, and the temperature T_∞ equal to 21·5°C.

4. MOTION OUTSIDE THE BOUNDARY LAYER

The motion of air outside the boundary layer and also the influx of air to the boundary layer have a character of unsteady flow.

During an experiment approximately twenty

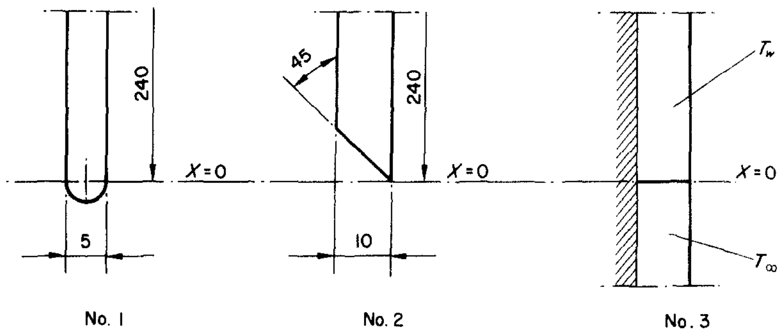


FIG. 2. Leading edge geometry of the investigated plates.

pictures were taken. Each of them shows a slightly different pattern of trajectories of small dust particles, which are nearly identical with streamlines. The unsteady character of the inflow of the air to the boundary layer is particularly noticeable in the region of the leading edge. Also in this region each change in the pattern of inflow causes significant changes in the boundary-layer region, because of the low velocities in the boundary layer. It is illustrated by Fig. 3(a, b, c) and 3(d, e), which show subsequent photographs of the area near the leading edge of No. 1 plate taken at 30-s intervals with the time exposure of about 10 s. The photographs show a visible change of the flow pattern. Moreover, it was observed that if on one side of the plate the flow pattern outside the boundary layer was, say, as in Fig. 3(a), on the other side of the plate the pattern of Fig. 3(e) was seen. However, in the case of the flow pattern shown in Fig. 3(d), or, in bigger scale, in Fig. 4, the flow was symmetrical on both sides of plate. Farther away from the leading edge a change in the pattern of the inflow causes less significant changes in the boundary region where the velocities are higher. Although the changes of pattern are less visible, they do take place as shown in Fig. 5(a, b). The photographs in Figs. 3–5 present the free convection flow around the No. 1 plate but the same change of pattern of flow was observed for the plate No. 2 and even No. 3. Figure 6 shows schematically the pattern of flow and nomenclature used in Fig. 7(a–e). For No. 1 plate beside the symmetrical flow, two extreme patterns are presented. For plate No. 2 and No. 3 only data from "symmetrical" pattern of flow are presented. For plate No. 2 "symmetrical" means a flow without velocity component $-v$ below the plate in plane of investigated flat surface. For plate No. 3 the term "symmetrical" is used for a chosen pattern considered as average for the given geometry. Figure 7 shows the velocity distribution for several values of x-coordinate. The points represent the experimental data and the lines the theoretical solutions, the plain line the boundary-layer solution (B.L.) and the chain line the first order perturbation solution (F.O.P.). The broken lines represent the outside-potential flow (P.F.) described from an approximate function of the line source strength shown in Fig. 8.

5. LEADING EDGE INFLUENCE

As mentioned before, the leading edge influence appears in two forms: (1) free convection starts on the leading edge surface which precedes the origin of the isothermal vertical plane, and (2) heat conduction of air causes that the temperature boundary layer starts upstream of the geometrical origin of the plate.

As the result of both these forms of the influence of the leading edge the air acquires a certain initial velocity as it approaches the neighbourhood of the flat isothermal surface

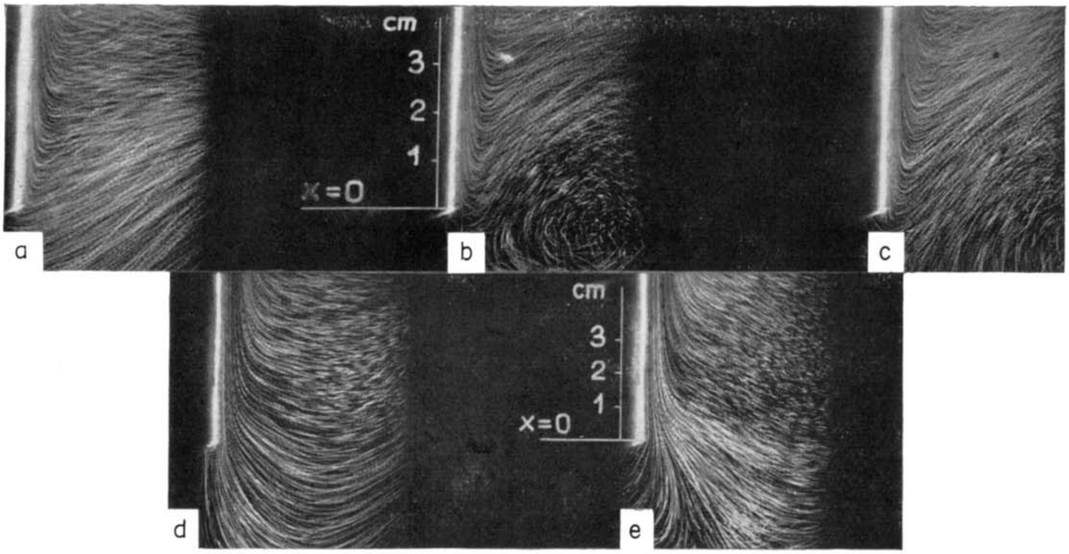


FIG. 3. Flow pattern near leading edge of No. 1 plate. The photographs of small dust trajectories (nearly streamlines) were taken in about 30-s intervals (a–e).

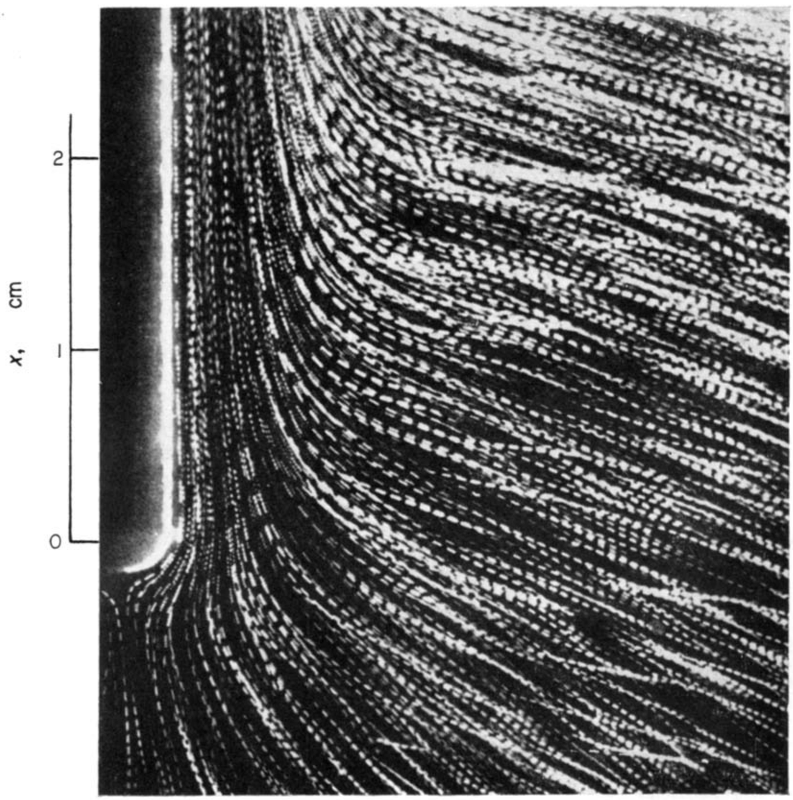


FIG. 4. Flow near the leading edge, symmetrical to the plane of No. 1 plate (similar to Fig. 3d).

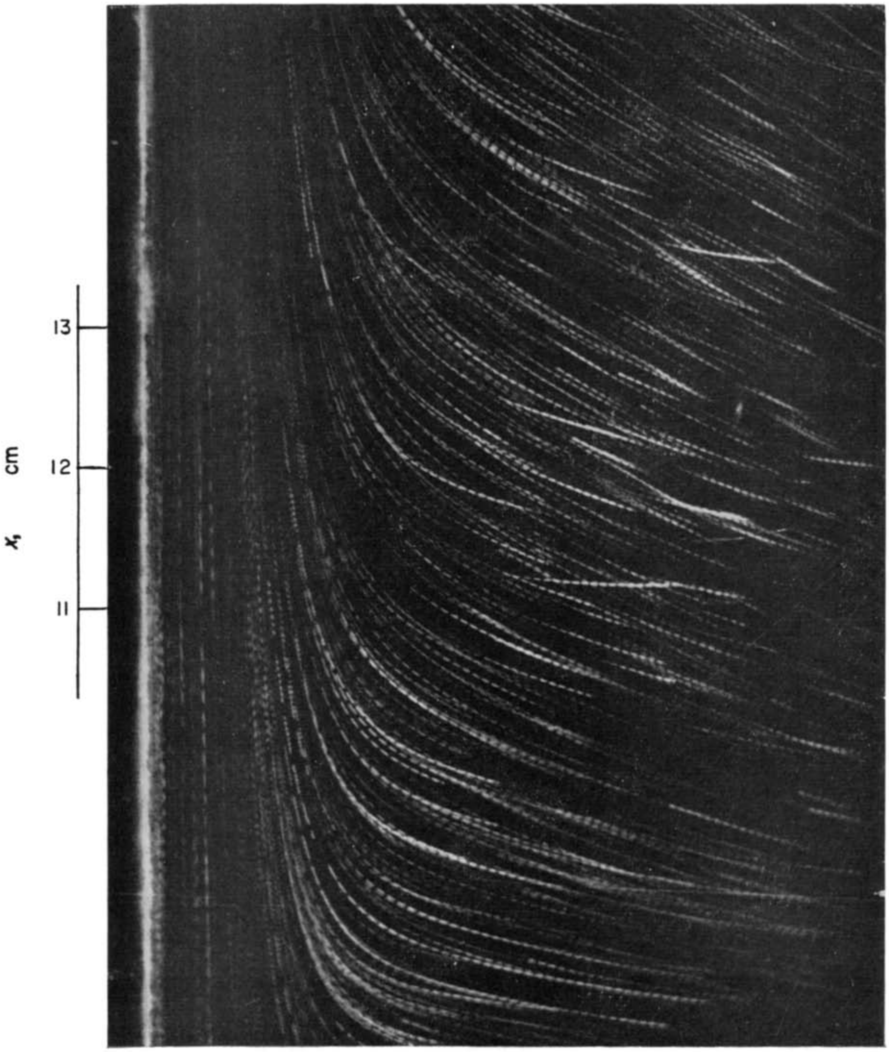


FIG. 5(a). Flow pattern farther away from the leading edge of No. 1 plate 4a—"symmetrical", 4b.

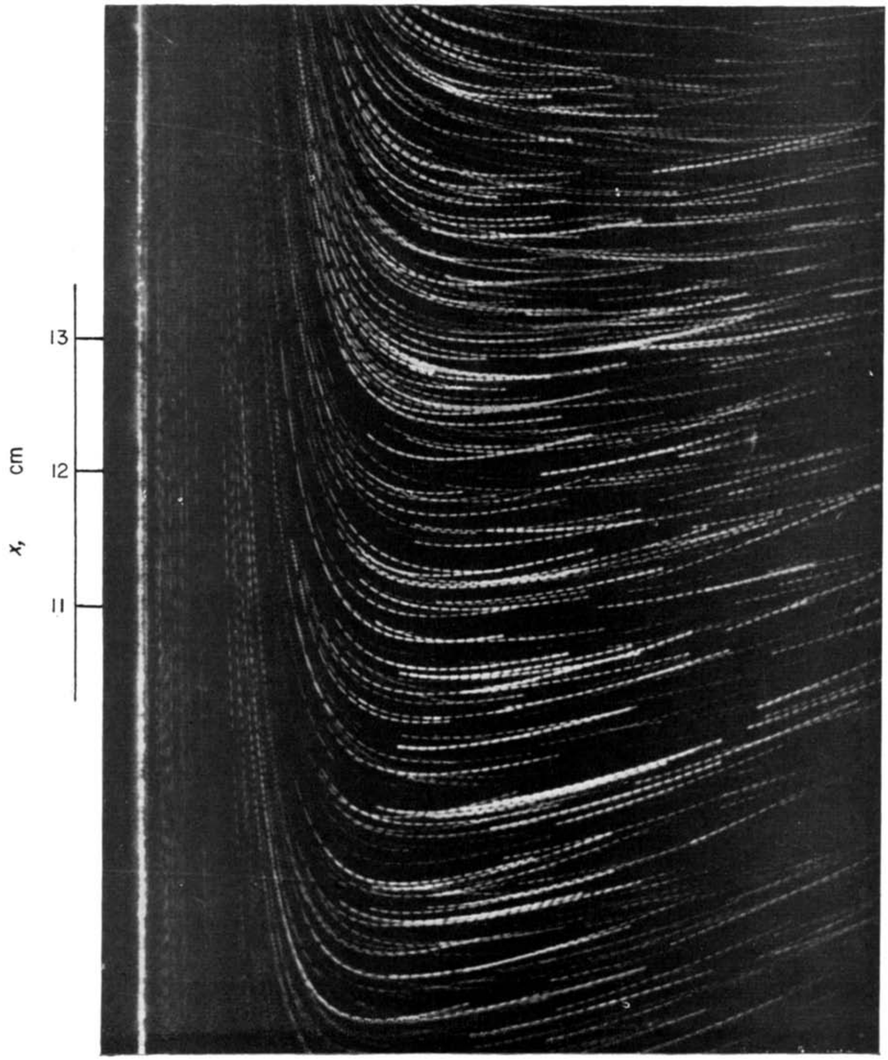


FIG. 5(b)

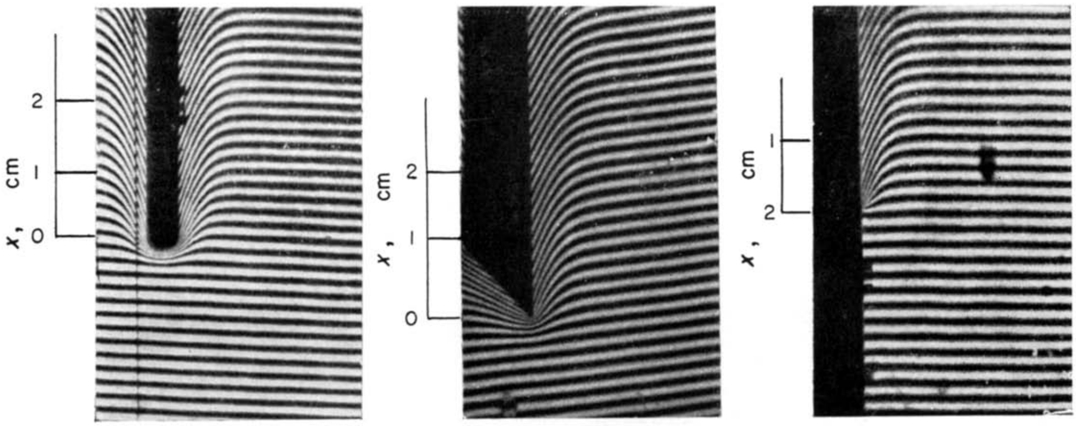


FIG 9. Interferograms showing temperature distribution near the leading edges.

where the main part of the convection phenomenon takes place.

The free convection effects of the leading edge surface may be eliminated by decreasing the size of this surface as in the case of No. 2 plate, or by additional cooling of this surface, as in the case of No. 3 plate. The symmetrical No. 1 plate has the leading edge surface very small as compared with the plates used in other investigations [2] and [3]. Yet the free convection on the leading edge surface of No. 1 plate is still noticeable. This is seen clearly in Fig. 9(b) which shows the velocity distribution at the x-coordinate equal zero, i.e. at the leading edge. In the case of No. 1 plate a velocity field exists with large values of the velocity gradient typical of viscous flow. The velocity fields around No. 2 and No. 3 plates show low gradients of velocity and therefore can be treated as cases of non-viscous flow.

Figure 9 presents the interferograms of the leading edge of the plates, on the basis of which was determined the temperature distribution in the plane of the plate near the leading edge, as shown in Fig. 10. The temperature distributions shown in Fig. 10 give an idea of the shift of the

Plate	u	v	Pattern of flow	Example
No. 1	○-	●-	(Symmetrical)	Fig 3d. Fig 4. Fig 5a
	♂	●	——	Fig 3a,b,c.
	♀	●	——	Fig 3c.
No. 2	▽	▼	(Symmetrical)	——
No. 3	□	■	(Symmetrical)	——

FIG. 6. Flow pattern and nomenclature of diagrams.

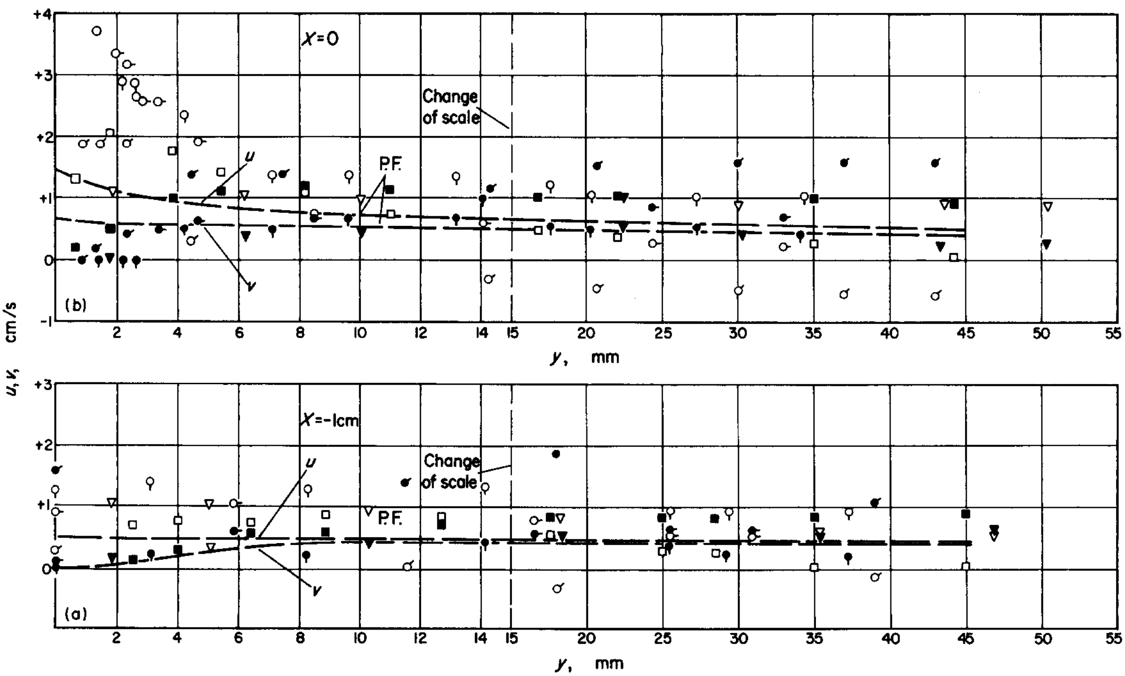
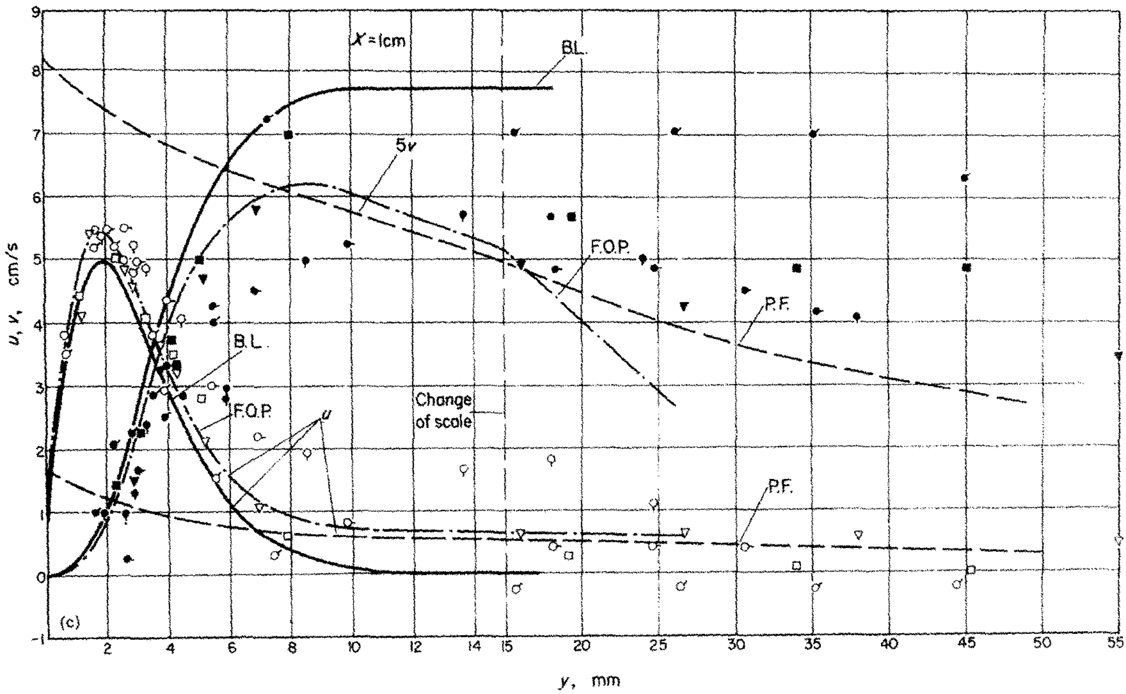


FIG. 7. Velocity distribution for the values of x-coordinate: Fig. 7 (a), $x = -1$ cm; Fig. 7 (b) $x = 0$; Fig. 7 (c), $x = +1$ cm; Fig. 7(d), $x = +2$ cm; Fig. 7(e), $x = +3$ cm; Fig. 7(f), $x = 12$ cm. B.L.—Boundary-layer solution; F.O.P.—First order perturbation solution; P.F.—Potential flow described by means of source shown in Fig. 8. *(continued overleaf)*



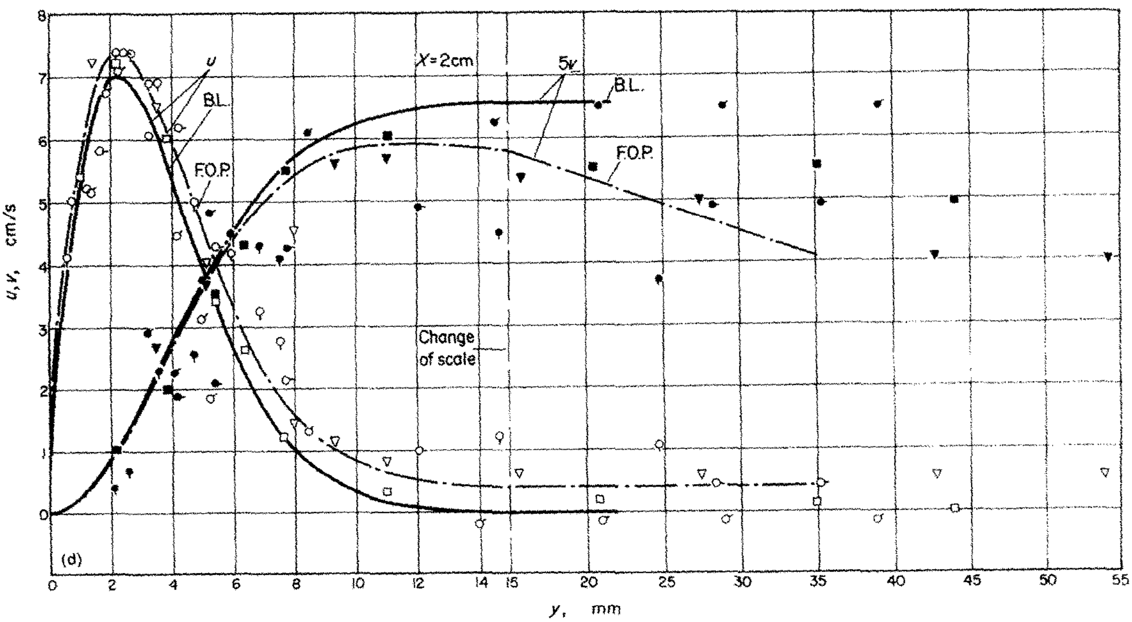
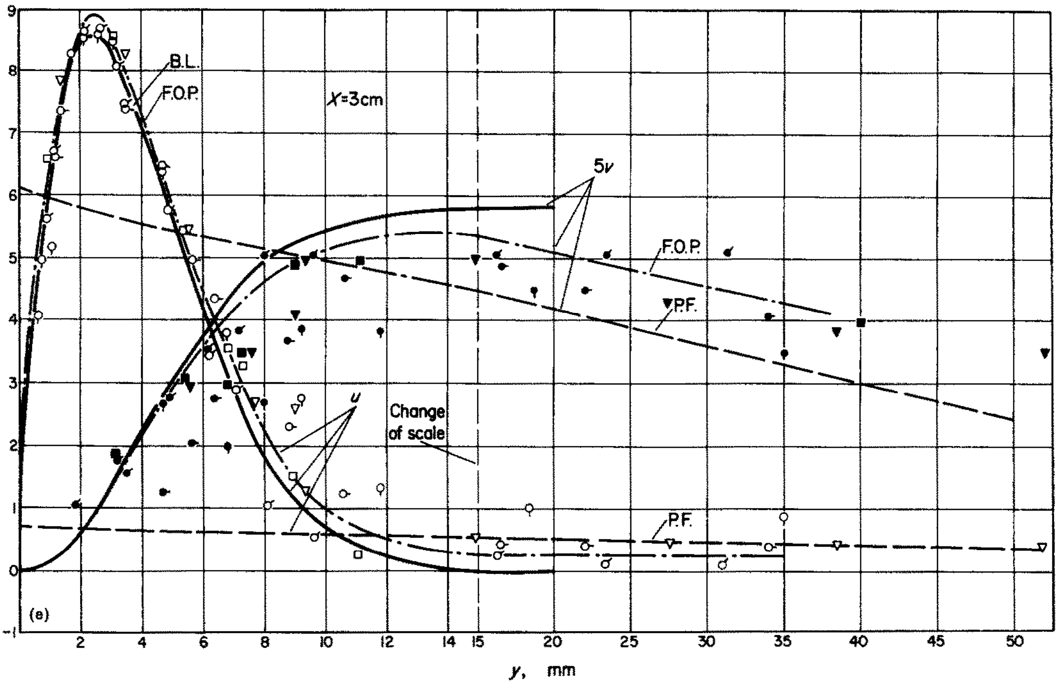


FIG. 7—*continued.*



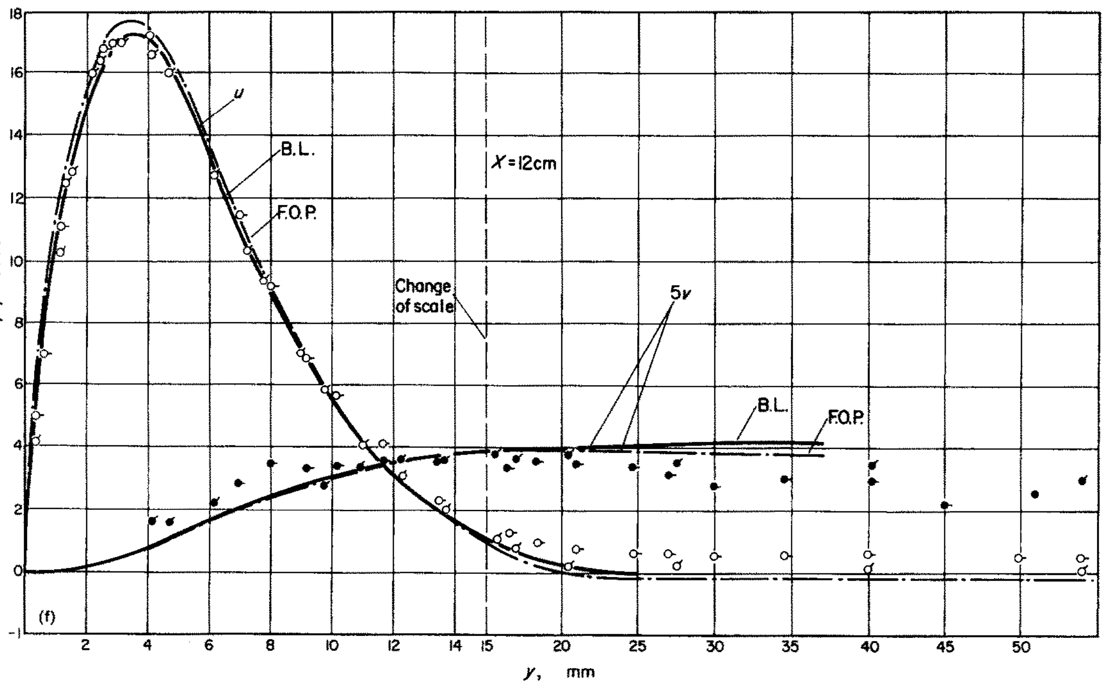


FIG. 7—*continued.*

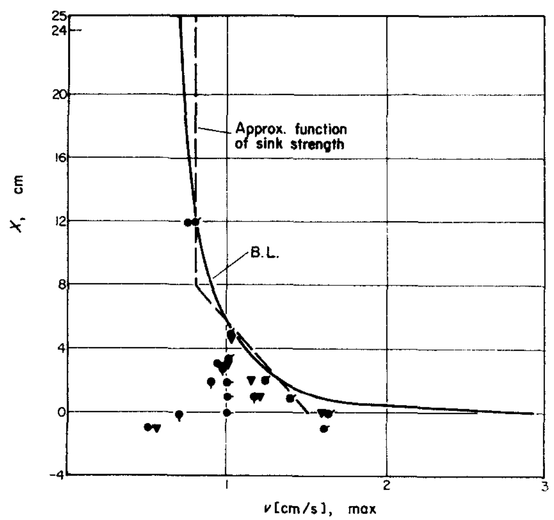
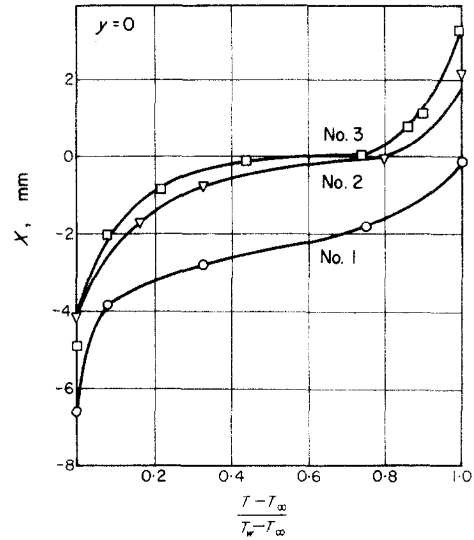


FIG. 8. Maximum of velocity component-v-distribution and strength of sink for describing outside flow.



$\frac{T - T_\infty}{T_w - T_\infty}$

FIG. 10. Temperature distribution in the plane of the plate near the leading edge.

boundary layer used in the integral analysis [5] and used for the correction of the experimental data of [2] and repeated in [3]. On the extension below the origin of the plane of the plate, the boundary conditions $u = 0$, $v = 0$ are not fulfilled, and for this reason the correction of the shift in the boundary layer is of a formal character only.

The heat conduction of the air is insignificant, which is confirmed by the experiments with No. 2 and No. 3 plates.

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Résumé—Souvent dans les articles sur la convection naturelle, les distributions de vitesse et de température au voisinage d'une surface plane verticale et isotherme sont comparées avec les previsions théoriques obtenues à partir de la solution des équations différentielles pour une plaque plane semi-infinie.

On essaie dans ce travail d'expliquer les différences qui ressortent de telles comparaisons et de montrer que le manque d'accord observé est dû au conditions sous lesquelles les expériences sont conduites.

On a donné une attention particulière à l'écoulement en dehors de la couche limite et à l'influence du bord d'attaque sur l'écoulement à l'intérieur de la couche limite.

Zusammenfassung—In Arbeiten über freie Konvektion wird oft die Geschwindigkeits- und Temperaturverteilung nahe einer ebenen, senkrechten isothermen Oberfläche verglichen mit theoretischen Lösungen von Differentialgleichungen, die für halbunendliche senkrechte Platten gewonnen wurden.

Hier wird der Versuch unternommen, die Unterschiede, die sich aus solchen Vergleichen ergeben zu erklären und zu zeigen, dass die beobachtete mangelnde Übereinstimmung auf die Bedingungen zurückzuführen sind unter denen die Versuche durchgeführt wurden.

Besondere Beachtung wurde der Strömung ausserhalb der Grenzschicht und dem Einfluss der Anströmkante auf die Strömung in der Grenzschicht geschenkt.

Аннотация—Экспериментальное распределение скоростей и температур у поверхности плоской изотермической пластины обычно сравнивается с теоретическим, полученным из решения уравнения, описывающего свободную конвекцию на вертикальной изотермической стенке.

В статье делается вывод, что расхождение между теоретическими и экспериментальными значениями, можно объяснить условиями проведения экспериментов.

Особое внимание уделяется течению во внешнем потоке, а также влиянию передней кромки на течение внутри пограничного слоя.